

Hong Kong Mathematics Olympiad (2006 – 2007)

Heat Event (Group)

香港数学竞赛 (2006 – 2007)

初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

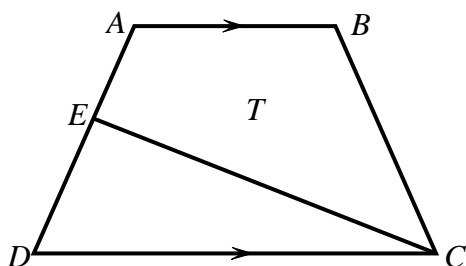
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 若由 1 至 50 内与 50 互质的整数有  $N$  个，求  $N$  的值。(注：正整数  $a$  与  $b$  称为互质若  $a$  与  $b$  的最大公因子是 1。)

If there are  $N$  integers from 1 to 50 that are relatively prime to 50, find the value of  $N$ . (Remark: positive integers  $a$  and  $b$  are said to be relatively prime if their greatest common divisor is 1.)

2. 如图一，在梯形  $ABCD$  中， $AB \parallel CD$ ， $\angle BCE = \angle ECD$ ， $CE \perp AD$  及  $DE = 2AE$ 。若  $\triangle DEC$  的面积是  $2007 \text{ cm}^2$  及四边形  $ABCE$  的面积是  $T \text{ cm}^2$ ，求  $T$  的值。

In Figure 1,  $ABCD$  is a trapezium,  $AB \parallel CD$ ,  $\angle BCE = \angle ECD$ ,  $CE \perp AD$  and  $DE = 2AE$ . If the area of  $\triangle DEC$  is  $2007 \text{ cm}^2$  and the area of quadrilateral  $ABCE$  is  $T \text{ cm}^2$ , find the value of  $T$ .



图一

Figure 1

3. 已知  $a^2 - 3a + 1 = 0$ 。若  $A = \frac{2a^5 - 5a^4 + 2a^3 - 8a^2 + 7a}{3a^2 + 3}$ ，求  $A$  的值。

Given that  $a^2 - 3a + 1 = 0$ . If  $A = \frac{2a^5 - 5a^4 + 2a^3 - 8a^2 + 7a}{3a^2 + 3}$ , find the value of  $A$ .

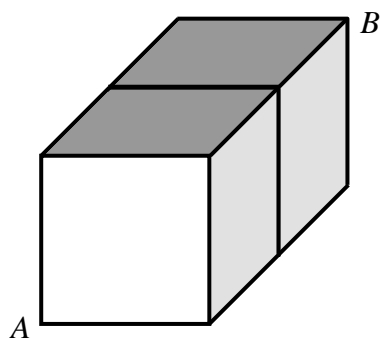
4. 已知点  $A$ 、 $B$  及  $C$  的坐标分别为  $(3, 4)$ 、 $(6, -4)$  及  $(8, 10)$ 。 $M$  及  $N$  分别为  $AB$  及  $BC$  的中点。 $X$  为  $AN$  上一点使得  $AX: XN = 2: 1$ 。若  $r = \frac{CX}{XM}$ ，求  $r$  的值。

Given that the coordinates of the points  $A$ ,  $B$  and  $C$  are  $(3, 4)$ ,  $(6, -4)$  and  $(8, 10)$  respectively.  $M$  and  $N$  are the midpoints of  $AB$  and  $BC$  respectively.  $X$  is a point on  $AN$  such that  $AX: XN = 2: 1$ .

If  $r = \frac{CX}{XM}$ , find the value of  $r$ .

5. 如图二，两个边长为  $1\text{ cm}$  的正方体组成一个  $1\text{ cm} \times 1\text{ cm} \times 2\text{ cm}$  的长方体。一只蚂蚁沿着长方体爬行，其爬行路线须为正方体的棱。牠从顶点  $A$  出发，以每分钟爬行  $1\text{ cm}$  的速度，于 4 分钟后到达顶点  $B$ 。若蚂蚁可行路线数目共有  $S$  个，求  $S$  的值。

In Figure 2, a  $1\text{ cm} \times 1\text{ cm} \times 2\text{ cm}$  rectangular box is made up by two cubes with side length  $1\text{ cm}$ . An ant is climbing along the box in a way that it must stay on the edges of the cubes through out the climbing. Starting from vertex  $A$  and climbing with a speed of  $1\text{ cm}$  per minute, it reaches vertex  $B$  after 4 minutes. If the total number of possible paths taken by the ant is  $S$ , find the value of  $S$ .



图二  
Figure 2

6. 若以 5 除  $7^{2007}$  所得的余数是  $R$ ，求  $R$  的值。

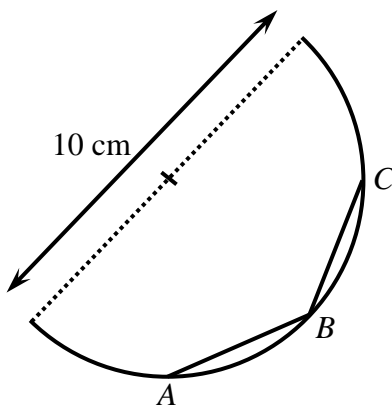
If the remainder of  $7^{2007}$  when dividing by 5 is  $R$ , find the value of  $R$ .

7. 设  $k = \sin 30^\circ + \cos 60^\circ + \sin 90^\circ + \cos 120^\circ + \cdots + \sin 1890^\circ + \cos 1920^\circ$ ，求  $k$  的值。

Let  $k = \sin 30^\circ + \cos 60^\circ + \sin 90^\circ + \cos 120^\circ + \cdots + \sin 1890^\circ + \cos 1920^\circ$ , find the value of  $k$ .

8. 如图三，已知半圆的直径为 10 cm。A、B 和 C 是半圆上任意的三点使 B 在弧 AC 上。设  $x$  为线段 AB 及 BC 的长度之和，求  $x$  可取的最大值。

In figure 3, given that the diameter of the semicircle is 10 cm. A, B and C are three arbitrary points on the semicircle where B is on the arc AC. If  $x$  is the sum of the length of the line segments AB and BC, find the greatest possible value of  $x$ .



图三

Figure 3

9. 在坐标平面上，点  $A = (-6, 2)$ 、 $B = (-3, 3)$ 、 $C = (0, n)$  及  $D = (m, 0)$  组成一个四边形 ABCD。求  $n$  的值使得该四边形 ABCD 的周界为最短。

In the coordinate plane, the points  $A = (-6, 2)$ ,  $B = (-3, 3)$ ,  $C = (0, n)$  and  $D = (m, 0)$  form a quadrilateral ABCD. Find the value of  $n$  so that the perimeter of the quadrilateral ABCD is the least.

10. 已知整数  $x$  及  $y$  满足  $3x + 5y = 1$ 。若  $S = x - y$  及  $S > 2007$ ，求  $S$  可取的最小值。

Given that integers  $x$  and  $y$  satisfying the equation  $3x + 5y = 1$ . If  $S = x - y$  and  $S > 2007$ , find the least possible value of  $S$ .